Qabalistic Points of View - Part 4 of a series

Pythagoras' Theorem and the Divine Proportion

By Michael Freedman

"Geometry has two great treasures: one is the Theorem of Pythagoras; the other is the division of a line into extreme and mean ratio, which is the Golden Section that reveals the Divine Proportion.

"The first we may compare to a measure of gold; the second we may name a precious jewel."

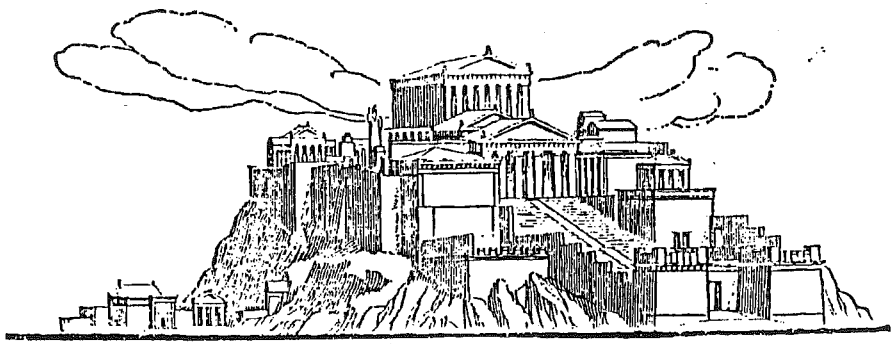
Johannes Kepler, astronomer & astrologer, 1571-1630.

AS STUDENTS OF HIGH MAGIC, WE seek to be aware of the unity and harmonies underlying all the differences we perceive between one thing and another in life. Vital to this process is awareness of the graceful harmonies and elegant proportions which exist in the shapes and sounds of life.

The masters of the ancient mysteries of science, examined reality principally through the metaphors of Sacred Geometry and Sacred Harmony.

Music is the study of the proportional relationships of sound frequencies, while Geometry is the study of the laws of spatial proportions and relationships. For example, the science of musical harmony is practically identical with the science of symmetry in crystals. In this series, we are concentrating on only one of these two great studies, Sacred Geometry.

Johannes Kepler has described Pythagoras' Theorem and the Divine Proportion as the two great treasures of Geometry.



PART I:
THE THEOREM OF PYTHAGORAS

The Theorem of Pythagoras, named for the 6th century b.c.e. Greek sage, states that:

In a right-angled triangle,
the square on the hypotenuse
is equal to the sum of the squares
on the other two sides.

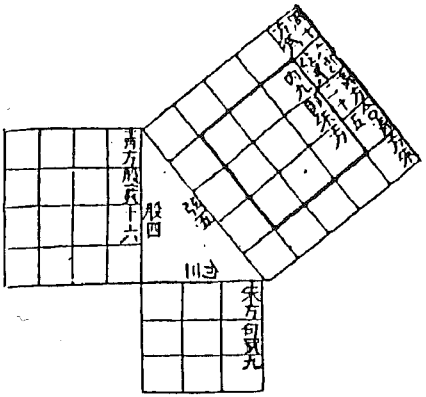
The hypotenuse is the longest side, and is opposite to the right angle.

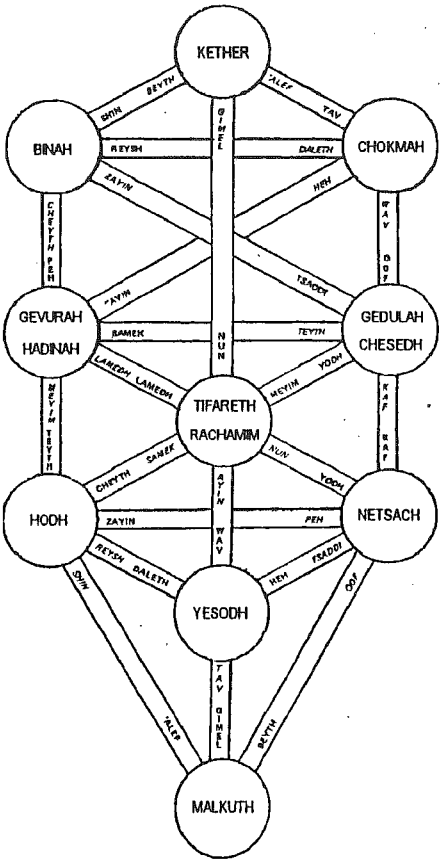
Older than Pythagoras

Although this theorem has been called the Theorem of Pythagoras in the Western world since the Greek teacher of geometry Euclid gave it that name in 300 b.c.e., it has been part of human knowledge from long before the days of Pythagoras throughout the whole world for thousands of years throughout the world.

The ancient Egyptians used their knowledge of the Theorem to construct wooden triangles whose sides were 3, 4 and 5 cubits long, thus giving them a right angle for use in building their palaces, temples and pyramids.

The figure below shows an illustration of the theorem taken from an ancient Chinese geometrical treatise.





The Tree of Life

There are many examples of right- angled triangles to be found in any Tree of Life diagram. For example, look at the Tree of Life diagram above. It is referred to Kether in "Assiyah among the 37 Trees of Life in Weavers' Wood. It is one of the four Trees of Life in Weavers' Wood that include all the paths of the Lightning Flash; and is hence used meditatively to ascend and descend through the whole of the Tree. It is possible to see twenty right-angled triangles in it.

We can use the Theorem of Pythagoras to calculate the length of any path between any pair of Sefiroth in any Tree of Life.

In the previous article in this Series, we saw how the Tree of Life emerges from a series of overlapping circles of equal size. For mathematical purposes, let us define the radius of the World Circles as one "Tree of Life Cubits" in length. Let us call it a Tolcubit.

It can be seen in the Tree of Life illustrated that the six Paths from Kether to Chokmah to Chesedh to Rachamim to Hadinah to Binah to Kether are equal to the radius of the World Circles, i.e., 1 tolcubit in length.

The paths from Kether to Rachamim, Chokmah to Hadinah and Binah to Chesedh are each equal to the diameter of a World Circle, i.e., 2 tolcubits long.

Consider the right-angled triangle formed by the paths linking Chokmah, Binah and Rachamim, of which the hypotenuse is the path from Binah to Rachamim [2 tolcubits]. The path from Chokmah to Chesedh is 1 tolcubit in length.

Therefore, the third path, from Chokmah to Binah can be calculated by Pythagoras' Theorem to be the square root of 3.

$$\sqrt{[2^2 - 1^2]} = \sqrt{[4 - 1]} = \sqrt{3}.$$

The length of the radius of the World Circles is described by some occultists as an 'Alef unit. It could be expressed in any unit, inches, metres, miles or parasangs, if you like.

Using the same method, it is possible to work out the length of every path able to be drawn in any Tree of Life. In fact, all the possible Paths must be one of five lengths. The following table sets out enough examples for you to be able to check this for yourself.

Path	Tolcubits
Kether to Chokmah	1
Kether to Chesedh	$\sqrt{3}$
Chokmah to Binah	$\sqrt{3}$
Kether to Rachamim	2
Kether to Netsach	$\sqrt{7}$
Binah to Netsach	$\sqrt{7}$
Malkuth to Binah	$\sqrt{13}$

Students of the Qabalah will know how important the numbers 3, 7 and 13 are as symbols of the Unity underlying the manifest multiplicity of existence.

If we add together the lengths that the Paths can be $[1 + 2 + \sqrt{3} + \sqrt{7} + \sqrt{13}]$, their sum approximates very closely to 11. The first seven prime numbers are 1, 2, 3, 5, 7, 11 and 13.

The Number Five not in the Tree of Life

Each of the first seven primes are presented, one way and another, in the fundamental structure of the Tree of Life, except one, the prime number Five.

Although the number 5 does not appear anywhere in the Tree of Life, it is present and predominant in the other great treasure of Geometry, according to Kepler, the Golden Section or Divine Proportion. The Divine Proportion appears nowhere in the Tree of Life, but it is found in the Five-pointed star we call the Pentagram.

PART II

THE DIVINE PROPORTION

If the principal glyph in the Qabalah is the Tree of Life, the principal glyph in High Magic is the Pentagram. The Pentagram is often called the Pentalpha, because it looks as though it is made up of five As.

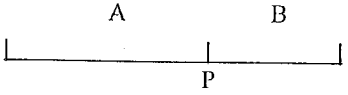
Phi, the name of the Divine Proportion

The mathematical symbol for the Divine Proportion, or Golden Section, is the Greek letter Phi ϕ. Among the basic correspondences of the Sacred Alphabets, the letter Phi is referred to the number 500 and to the Element of Spirit.

The Golden Section

The formal geometrical description of the Golden Section is as follows:

If a straight line is cut at a point P into two sections, so that the ratio of A to B is the same as the ratio of the whole line to A, the division is described as the Golden Section



This can be expressed as the Divine Proportion: $\frac{A}{B} = \frac{A+B}{A}$

The Value of Phi

If section A of the whole line is regarded as equal to 1 unit, then the above ratios can be expressed as:

$$\frac{1}{B} = \frac{1+B}{1}$$

The only possible mathematical solution to this equation is:

$$\frac{1}{0.6180339...} = \frac{1+0.6180339.....}{1}$$

Like many other important mathematical constants, such as the ratio of the diameter of a circle to its diameter Pi [π = 3.14159...], Phi ϕ cannot be expressed as an exact number.

Leonardo Fibonacci of Pisa

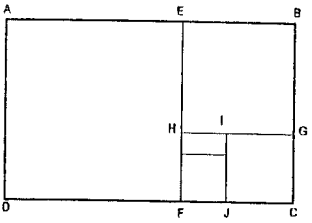
Increasingly accurate approximations to the value of Phi, 1.618034, can be found in the ratios of any two successive numbers in the remarkable Fibonacci series of numbers.

The Fibonacci series of numbers was named after the 12th Italian mathematician, Leonardo Fibonacci of Pisa, who also introduced the symbol for zero, 0, into European mathematics, after study as a boy with an Arab master of mathematics in North Africa.

In the Fibonacci series, the first two terms can be any pair of numbers; and each succeeding term is the sum of the two numbers immediately preceding it. For example:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, and so ....

Let us look at the ratios in this sequence of numbers, and see how the ratio is first below and then above the Divine proportion in each pair, but is getting closer and closer to it with each succeeding pair.

1/1	=	1.0000	2/1	=	2.0000
3/2	=	1.5000	5/3	=	1.6667
8/5	=	1.6	13/8	=	1.6250
21/13	=	1.615385	34/21	=	1.619048
55/34	=	1.617647	89/55	=	1.618182
144/89	=	1.617978	233/144	=	1.618055
377/233	=	1.618026	610/377	=	1.618037

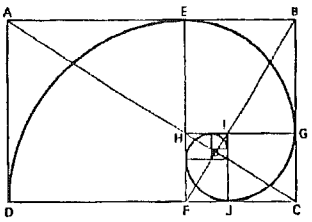


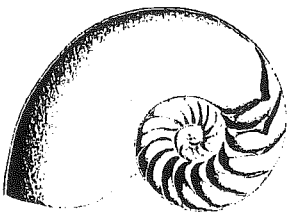
The Golden Rectangle

A Golden Rectangle is one such that is drawn so that, if its shorter side is equal to one unit, then its longer side is equal to Phi, i.e., 1.618 units.

In the diagram above, let the side AB = 1.618, then the side AD = 1.000. If you then mark off a square ADEF with sides of 1 unit within the rectangle, the remainder EBCF is also a Golden Rectangle. This process can be repeated ad infinitum, as is shown in the diagram above.

The equivalent corner in each successive square in such a series of divisions of the rectangle, DEGJ, etc., all lie on a curve known as a Logarithmic Spiral, as in the next diagram.

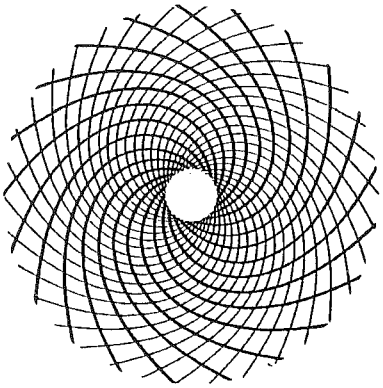




The Divine Proportion in Nature

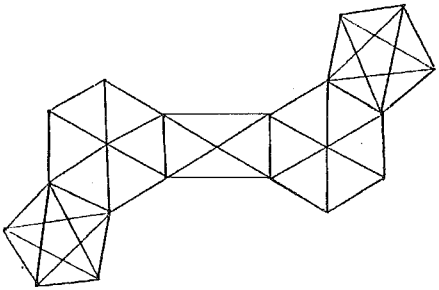
The Logarithmic Spiral, which is based on the Divine Proportion, appears in many places in nature, often at a very fundamental level.

A well known example is the Nautilus shell illustrated above. Note its similarity to the curve in the previous diagram.



Another example is the spiral pattern formed by the seeds of the Sunflower, a diagram of which is shown here.

The molecular structure of each step of the DNA spiral is a pentagram and a hexagon joined together, linked to another pentagram and hexagon by a Golden Rectangle.



Each step of the DNA spiral is linked to the step above and below it by a phosphate structure in the form of a square cross.

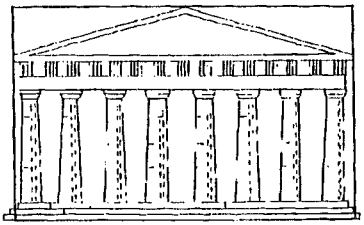
There are many other natural structures which include the golden ratio, and other figures of symbolic importance in the sphere of High Magic.

The Divine Proportion in Art

During the 19th century, the German psychologist Gustav Fechner conducted an experiment that has often been repeated since by other psychologists.

He showed a number of rectangles, ranging in shape from square to extremely narrow to a large number of people and asked them to select one rectangle which they considered to have the most satisfying , harmonious or pleasing shape.

The vast majority of his subjects chose rectangles that were either the Golden Rectangle or very close to it in shape. Noone knows why this shape should be aesthetically pleasing to human beings. Fechner did not discover this but merely provided experimental evidence that confirmed it.



Architects and artists have incorporated the Divine Proportion, often in the form of the Golden Rectangle for thousands of years.

The proportions of the Acropolis which crowned the ancient Temple complex in Athens, one of the most beautiful buildings in the world, is formed from a series of golden rectangles. Leonardo da Vinci and many other artists have used the Divine proportion in their paintings, sculptures and architecture. ■